Inverse Scattering of Nonuniform, Symmetrical Coupled Lines

Tun-Ruey Cheng, Te-Wen Pan, and Ching-Wen Hsue, *Senior Member, IEEE*

***Abstract*— A novel technique is developed to reconstruct the physical layouts of nonuniform symmetric couplers from the scattering parameters at terminals. The coupler is decomposed into even- and odd-mode lines in which the decoupled modal lines are reconstructed from their respective scattering parameters. The effect of unequal modal propagation delays on the reconstructed coupler is also addressed. Numerical results are presented to illustrate the validity of this reconstruction technique.**

***Index Terms*— Coupler, inverse scattering.**

I. Introduction

Uniform coupled-line circuits have been used for many applications including filter, directional couplers, and impedance matching networks. These circuits are designed by utilizing the impedance, admittance, chain, and other parameters characterizing the coupled-line network. From the experimental viewpoints, we often characterize the coupler with a set of scattering parameters at terminal ports.

Several authors had paid a lot of attention to the analysis of coupled transmission lines [1]–[8]. Most of the work thus far has focused mainly on the formulation and computation techniques of scattering waves on transmission lines. Only a few papers [4], [5] were concerned with the synthesis problems in which the structures of coupled lines are obtained from scattering parameters. In particular, they [4] employed equal modal propagation delays to simplify the mathematical formulation. In this letter, we use a nonuniform line reconstruction technique [6] to reconstruct a nonuniform, symmetric coupler when scattering parameters at terminal ports are given. The effect of unequal modal propagation delay on the reconstructed line is also addressed.

II. Theory

We consider the wave scattering for a signal incident upon the coupler from port 1 in Fig. 1. The appropriate wave to analyze the responses of symmetric coupled transmission lines is to split the input signal into even and odd modes [8]. We then can express the scattering parameters at the left-hand side of the coupled lines as

$$\mathrm{S}_{11}(\omega) = \tfrac{1}{2}\,\Gamma_e(\omega) + \tfrac{1}{2}\,\Gamma_o(\omega) \quad \text{(1a)}$$

$$\mathrm{S}_{21}(\omega) = \tfrac{1}{2}\,\Gamma_e(\omega) - \tfrac{1}{2}\,\Gamma_o(\omega) \quad \text{(1b)}$$

where

ω	angular frequency;
$\mathrm{S}_{11}(\omega)$	reflected coefficient at port 1;
$\mathrm{S}_{21}(\omega)$	coupling wave propagating in the backward direction coefficient at port 2;
$\Gamma_e(\omega)$, $\Gamma_o(\omega)$	even- and odd-mode reflection coefficients, respectively.

Note that (1) is valid when both ports 3 and 4 are terminated with matched loads. As long as the structure is symmetric, (1) is valid regardless of the physical layouts of a two-line coupler. Because a nonuniform coupler can be treated as cascaded multiple-section coupled lines, this, in turn, indicates that (1a) and (1b) are applicable to a nonuniform, symmetric coupler. By inverting (1a) and (1b), we can express $\Gamma_e(\omega)$ and $\Gamma_o(\omega)$ in terms of the scattering parameters

$$\Gamma_e(\omega) = \mathrm{S}_{11}(\omega) + \mathrm{S}_{21}(\omega) \quad \text{(2a)}$$

$$\Gamma_o(\omega) = \mathrm{S}_{11}(\omega) - \mathrm{S}_{21}(\omega). \quad \text{(2b)}$$

Equation (2) reveals that the even-mode reflection coefficient is the sum of $\mathrm{S}_{11}(\omega)$ and $\mathrm{S}_{21}(\omega)$, while the odd-mode reflection coefficient is the difference of $\mathrm{S}_{11}(\omega)$ and $\mathrm{S}_{21}(\omega)$. Note that both even- and odd-mode waves are independent/uncoupled signals in the decoupled transmission line configurations. In particular, we know that a nonuniform transmission line can be reconstructed from its reflection coefficient [6]. The reconstructed transmission line is represented by the space distribution of both characteristic impedance and propagation delay. By converting $\mathrm{S}_{11}(\omega)$ and $\mathrm{S}_{21}(\omega)$ into $\Gamma_e(\omega)$ and $\Gamma_o(\omega)$, we can reconstructed the even-mode line and odd-mode line of a coupler. In other words, we may obtain even-mode impedance profile $\hat{Z}_e(t)$ and odd-mode impedance profile $\hat{Z}_o(t)$ from the scattering parameters, where t is the time.

For a microstrip coupler, the relations between the physical layout of the coupler and characteristic impedances of two modal lines are given in [7]. We then can obtain the physical parameter $W(x)/H$ and $S(x)/H$ from the modal characteristic impedances, where W is the line width, H is the thickness of the substrate, S is the spacing between two lines, and x is the space variable in the direction of propagation. However, in order to obtain the physical parameters $W(x)/H$ and $S(x)/H$ from modal characteristic impedances $\hat{Z}_e(t)$ and $\hat{Z}_o(t)$, we need to convert time-domain impedance profiles $\hat{Z}_e(t)$ and $\hat{Z}_o(t)$ into modal profiles $Z_e(x)$ and $Z_o(x)$. The propagation delay and the coupled physical length are interrelated via modal velocities ν_e, ν_o and modal propagation delays τ_e, τ_o

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The authors are with the Department of Electronic Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, R.O.C.

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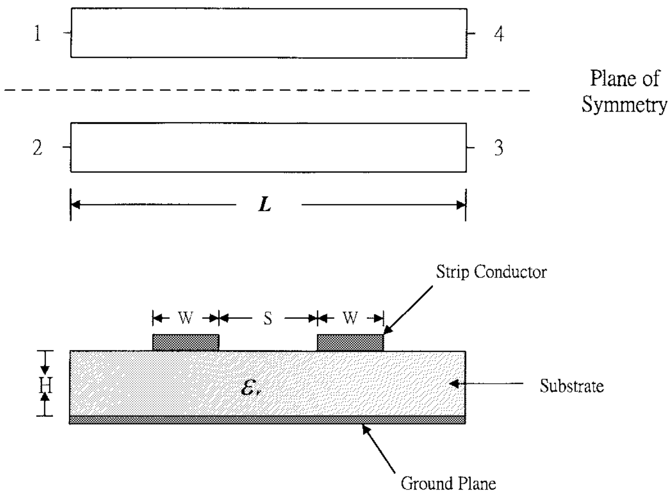

Fig. 1. Symmetrical coupled transmission lines.

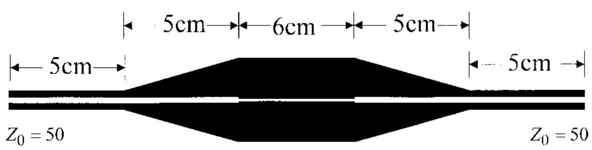

Fig. 2. The physical layout of nonuniform microstrip coupled lines.

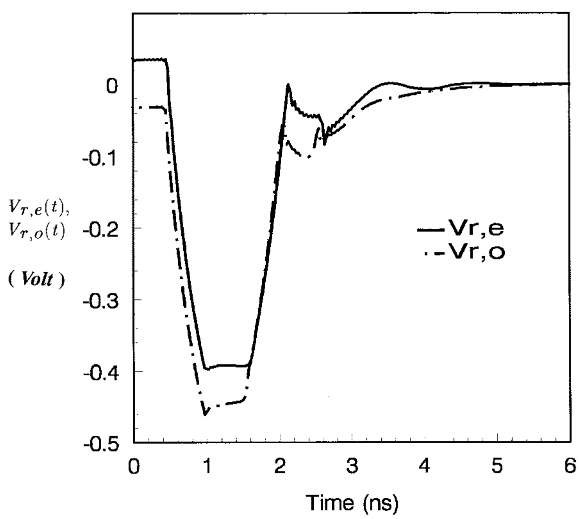

Fig. 3. The time-domain modal reflected waves of the coupler in Fig. 2.

as follows:

$$L_e = \nu_e \cdot \frac{\tau_e}{2} = \frac{c}{\sqrt{\epsilon_e}} \cdot \frac{\tau_e}{2} \tag{3a}$$

for the even mode

$$L_o = \nu_o \cdot \frac{\tau_o}{2} = \frac{c}{\sqrt{\epsilon_o}} \cdot \frac{\tau_o}{2} \tag{3b}$$

for the odd mode, where

L_e, L_o modal coupling lengths;
c speed of light in vacuum;
ϵ_e, ϵ_o effective dielectric constants for even and odd modes, respectively.

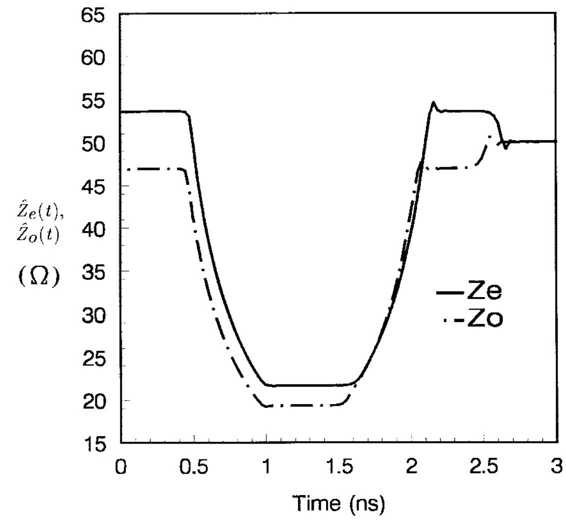

Fig. 4. The reconstructed modal impedance profiles in time domain.

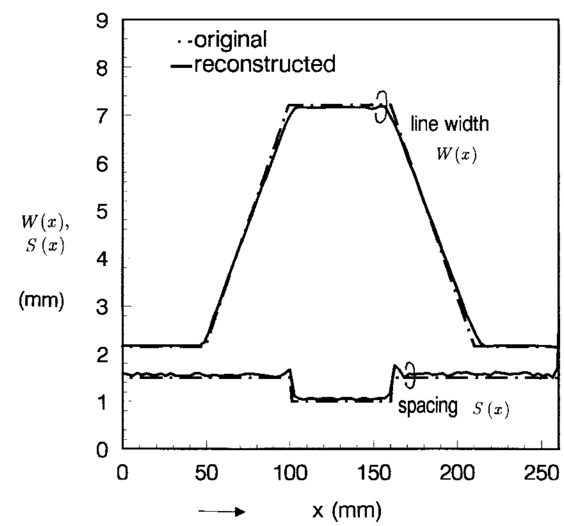

Fig. 5. The reconstructed physical layouts: $W(x)$ and $S(x)$.

Of course, the even-mode coupling length L_e should be equal to the odd-mode coupling length L_o. Both ϵ_e and ϵ_o are determined by the distributive capacitances of two modes. Those capacitances may be visualized as being made up of parallel-plate capacitances and appropriate fringing capacitances [7]. We, therefore, can obtain the physical parameters $W(x), S(x)$ of the coupler.

III. Numerical Results

Fig. 2 shows the layout of a nonuniform coupler, which is assumed to be built on a Duriod substrate having thickness 31 mils and relative dielectric constant $\epsilon_r = 2.5$. We get the scattering parameters $S_{11}(\omega)$ and $S_{21}(\omega)$ by using HP/EESOF CDS software simulator for the frequency range extending from dc to 18.045 GHz. To obtain the physical structure of the coupler, we first convert the scattering parameters into time-domain modal reflected wave $V_{r,e}(t)$ and $V_{r,o}(t)$. The modal reflected waves in Fig. 3 are obtained by using (2) and inverse Fourier transform. Note that both modal reflected waves have prolonged ripples which are due to internal reflection-transmission processes on nonuniform modal lines. By utilizing the reconstruction method [6], we then obtain the modal characteristic impedance profiles $\hat{Z}_e(t), \hat{Z}_o(t)$ of the coupler, which are shown in Fig. 4. Apparently, the even-mode wave has a longer total propagation delay than odd-mode wave. Fig. 5 shows the reconstructed line width W and spacing S as a function of x. Clearly, the reconstructed results are in good agreement with the original layouts of the coupler.

IV. Conclusion

We have developed a novel technique to reconstruct the layouts of nonuniform, symmetric couplers from the scattering parameters at terminal ports. By taking account of both internal transmission-reflection processes of waves and unequal modal propagation delays, we can accurately reconstruct a nonuniform coupler. This technique has useful applications to arbitrary waveform synthesis and signal filtering.

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